Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

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MAT 101
A Survey of Mathematical Reasoning

Final Exam

Instructor:

Name: Alexander Kasiukov
Office: Suffolk Federal Credit Union Arena, Room A-109
Phone: (631) 851-6484
Email: kasiuka@sunysuffolk.edu
Web Site: http://www.kasiukov.com

Please print the requested information in the spaces provided:

Student:

Name:	
Student Id:	
Email:	*include to receive the final grade via email* ONLY *if you are not getting email updates*

- *Notes and books are permitted on this exam.*
- *Graphing calculators, smartwatches, computers, cell phones and any other communication-capable devices are prohibited. Their mere presence in the open (even without use) is a sufficient reason for an immediate dismissal from this exam with a failing grade.*
- *You will not receive full credit if there is no work shown, even if you have the right answer. Please don't attach additional pieces of paper: if you run out of space, please ask for another blank final.*

Existential presupposition is in effect for *(some x are y)* statements. The proposition *(some cats are smart)* means that the universe of discourse contains at least one cat which is smart. Existential presupposition is not in effect for *(all x are y)* statements. The proposition *(all unicorns are mammals)* means that every unicorn existing in the universe of discourse must be a mammal, but it does not assume that even a single unicorn exists.

Problem 1. Suppose three suspects were caught in an art museum. Before they surrendered to police, they agreed that each one of them will tell a half-truth to their interrogators. They were separated and asked two questions: which painting they wanted to steal and who commissioned them.

- The first said that they wanted to steal the Rembrandt and the crime boss Big Joe promised to buy it.
- The second said that they came to get the Degas and stated that it was definitely not the Big Joe who sent them.
- The last suspect claimed that they came to steel the Monet, and were commissioned by the notorious Lucky Sam.

(1). Using disjunction to represent a half-true statement, we can write the information conveyed by the first suspect as

$$\text{Rembrandt} \vee \text{Joe}.$$

Assuming, in addition to the information gained in the interrogations, that the Rembrandt was not the target of the theft, formulate a valid argument in Gentzen's notation:

$$\begin{array}{l} \text{Rembrandt} \vee \text{Joe} \\ \neg\text{Rembrandt} \\ \hline ??? \end{array}$$

stating who commissioned the theft. For this problem, only the statement of the argument, rather than verification of its validity, is needed.

Space for your solution:

(2). Write the argument which you gave as the answer to the previous problem as a single statement of propositional logic. (Use any necessary logical connectors that may be implicit in Gentzen's notation.)

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(3). Use the truth tables to verify the validity of this argument.

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(4). Suppose we record the information gained in the interrogation in the form of the following Gentzen-style argument:

$$\frac{\begin{array}{l}\text{Rembrandt} \vee \text{Joe} \\ \text{Degas} \vee (\neg\text{Joe }) \\ \text{Monet} \vee \text{Sam}\end{array}}{???}$$

Assume that in addition to this information the police knows that the thiefs could only steal one painting, and that Big Joe and Lucky Sam are enemies, so the the thiefs could only serve one of them but not both. How can that additional information be added to the above assumptions?

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(5). Use the distributivity of conjunction with respect to disjunction:

$$A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C),$$

to extract the truth from the information depicted in the previous subproblem. Hint: you may find it convenient two write conjunction as multiplication, and disjunction as addition, so that the above information gets recorded as

$$\begin{array}{l} \text{Rembrandt} + \text{Joe} \\ \text{Degas} + (\neg\text{Joe}\ ) \\ \text{Monet} + \text{Sam} \\ \ldots \\ \hline ??? \end{array}$$

It may also be helpful to write the falsehood of a statement X as $X = 0$, and its truth as $X = 1$.

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Problem 2. Consider the following syllogism:

All mathematics classes are hard.
Some logic classes are not hard.

Some logic classes are not mathematics.

(1). Draw a Venn diagram showing the categories of this syllogism.

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(2). Express the assumptions of this syllogism graphically in the Venn diagram.

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(3). Give a reason why this syllogism is a valid argument, or provide a counterexample showing that it is invalid.

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(4). Express this syllogism in the form of a Gentzen-style argument of predicate logic.

Space for your solution: