Suffolk County Community College
Michael J. Grant Campus

Department of Mathematics

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MAT 103
Statistics I

Final Exam: Solutions and Answers

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Problem 1. University of California, Berkeley graduate division admitted 44% of male and 35% of female applicants in the Fall of 1973.

Noticing this apparent discrepancy, Eugene A. Hammel, then the Associate Dean of the Graduate Division,[1] asked Peter Bickel, then a professor of statistics at Berkeley, to analyze the data. The results of that analysis [2] became one of the most widely cited examples of the statistical phenomenon called *Simpson's Paradox*. In this problem, we explore this phenomenon and its ramifications.

The original paper by Bickel at al. does not contain the raw data on the individual departments, but the Data Science Discovery platform [3] has a data set covering all the 12,763 applicants from the original study. It obscures the specific department names, but identifies the six most popular departments by the department codes A, B, C, D, E and F. In this problem, we will focus only on those six departments, and — in the interest of time — we will further group them into two groups. The departments A and B will form the "easy-to-get-into" group, and departments C, D, E and F will make up the "hard-to-get-into" group. The effect of the Simpson's paradox becomes even more pronounced when only those six departments are considered.

(1). Based on the aggregated six-department data:

	Male	**Female**
Accepted	1,511	557
Rejected	1,493	1,278

compute and compare the conditional probabilities:

$$P(\text{Accepted}|\text{Male}) =$$

$$P(\text{Accepted}|\text{Female}) =$$

and determine if there has been a bias against women in graduate admissions.

Space for your solution:

$$P(\text{Accepted}|\text{Male}) = \frac{1,511}{1,511 + 1,493} = \frac{1,511}{3,004} \approx 50\%$$
$$P(\text{Accepted}|\text{Female}) = \frac{557}{557 + 1,278} = \frac{557}{1,835} \approx 30\%$$

These probabilities seem to suggest a bias against women.

[1] see Cari Tuna (2009) "When Combined Data Reveal the Flaw of Averages", A Wall Street Journal interview with Peter Bickel, https://www.wsj.com/articles/SB125970744553071829,

[2] Bickel, P. J., Hammel, E. A., and O'Connell, J. W. (1975) "Sex bias in graduate admissions: Data from Berkeley", Science, 187, 398–403, http://brenocon.com/science_1975_sex_bias_graduate_admissions_data_berkeley.pdf

[3] Berkeley's 1973 Graduate Admissions Dataset, Data Science Discovery, University of Illinois at Urbana-Champaign, https://discovery.cs.illinois.edu/dataset/berkeley/

(2). Graduate admission decisions are made by individual departments. In the attempt to "look for the responsible parties", Professor Bickel and his colleagues analyzed data for each of the 101 departments separately. We will use a much more coarse analysis, grouping the six most popular departments into two groups and analyzing the admissions data for those two groups.

Here is the statistics for the easy-to-get-into departments (those labelled as "A" and "B" in the Data Science Discovery dataset):

Easy	**Male**	**Female**
Accepted	$1,178$	106
Rejected	520	27

and for the hard-to-get-into departments (labelled "C", "D", "E" and "F" in the same dataset):

Hard	**Male**	**Female**
Accepted	333	451
Rejected	973	$1,251$

Compute and compare the conditional probabilities:

$$P(\text{Accepted}|\text{Male}) =$$

$$P(\text{Accepted}|\text{Female}) =$$

separately for the easy-to-get-into and hard-to-get-into departments.

Space for your solution:

For the easy-to-get-into departments:

$$P(\text{Accepted}|\text{Male}) = \frac{1,178}{1,178+520} = \frac{1,178}{1,698} \approx 69\%$$

$$P(\text{Accepted}|\text{Female}) = \frac{106}{106+27} = \frac{106}{133} \approx 80\%.$$

For the hard-to-get-into departments:

$$P(\text{Accepted}|\text{Male}) = \frac{333}{333+973} = \frac{333}{1,306} \approx 25\%$$

$$P(\text{Accepted}|\text{Female}) = \frac{451}{451+1,251} = \frac{451}{1,702} \approx 26\%.$$

(3). What overall conclusion can you draw from this analysis of admissions data? Did Berkeley discriminate against women in their fall 1973 graduate admissions?

Space for your solution:

The department type was a stronger predictor of admission than the sex of the applicant. Women were more likely to apply to the hard-to-get-into departments, while men disproportionately applied to the easy-to-get-into departments, thus the aggregation of the data from all six departments obscured the effect of the department choice on the admission outcomes, creating an illusion of a bias against women.

When the two department groups are analyzed separately, the effect of the department choice is separated from the effect of sex (and the bias *in favor of* women in the easy-to-get-into departments becomes apparent).

Department choice is the decision made by the applicant, not by the school. While it is entirely possible that women suffered from bias against them on the way leading them to their department selection, graduate admission statistics does not indicate any bias against women on the part of the school.

Problem 2. The 3rd Nerve Palsy causes the involved eye to deviate in "down and out" direction, and may result in partial or complete ptosis, otherwise know as "lazy eye". In 98% of cases, this condition is ischaemic (resulting from a restriction to blood supply) and the patients make full recovery without treatment. In 1% of cases, 3rd Nerve Palsy is caused by an aneurysm and in another 1% of cases — by cavernous sinus pathology (CSP). We will assume that these three conditions are always mutually exclusive.

(1). Untreated aneurysms are fatal in 2% of cases, and untreated CSPs — in 50% of cases. Determine the risk of a patient with 3rd Nerve Palsy dying from its cause, if left undiagnosed and untreated.

Space for your solution:

$$P\begin{pmatrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{pmatrix} = \boxed{\text{the formula of total probability}} =$$

$$= P\begin{pmatrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{ischaemia}\end{pmatrix} \cdot P\left(\begin{matrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{matrix}\,\middle|\,\begin{matrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{ischaemia}\end{matrix}\right)$$

$$+ P\begin{pmatrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{aneurysm}\end{pmatrix} \cdot P\left(\begin{matrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{matrix}\,\middle|\,\begin{matrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{aneurysm}\end{matrix}\right)$$

$$+ P\begin{pmatrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{CSP}\end{pmatrix} \cdot P\left(\begin{matrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{matrix}\,\middle|\,\begin{matrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{CSP}\end{matrix}\right) =$$

$$= 98\% \cdot 0 + 1\% \cdot 2\% + 1\% \cdot 50\% = 0.0052,$$

meaning that $5,200$ deaths will occur for every million of untreated 3rd Nerve Palsy cases, or approximately 1 in 192.

(2). A magnetic resonance angiogaphy (MRA) scan is a non-invasive test for detecting anuerusyms and CSP. However, MRA carries a 5% risk of non-detection.

Determine the risk of a patient with 3rd Nerve Palsy dying from its cause, after having an MRA and the appropriate treatment, if an aneurysm or CSP is detected by the MRA. Assume that the treatment prevents death from aneurysm with certainty, but a treated CSP patient still has 20% risk of death.

Space for your solution:

$$P\begin{pmatrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{pmatrix} = \boxed{\text{the formula of total probability}} =$$

$$= P\begin{pmatrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{ischaemia}\end{pmatrix} \cdot P\left(\begin{matrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{matrix} \,\middle|\, \begin{matrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{ischaemia}\end{matrix}\right)$$

$$+ P\begin{pmatrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{aneurysm}\end{pmatrix} \cdot P\left(\begin{matrix}\text{patient}\\ \text{has}\\ \text{undiagnosed}\\ \text{aneurysm}\end{matrix} \,\middle|\, \begin{matrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{aneurysm}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{matrix} \,\middle|\, \begin{matrix}\text{patient}\\ \text{has}\\ \text{undiagnosed}\\ \text{aneurysm}\end{matrix}\right)$$

$$+ P\begin{pmatrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{CSP}\end{pmatrix} \cdot P\left(\begin{matrix}\text{patient}\\ \text{has}\\ \text{undiagnosed}\\ \text{CSP}\end{matrix} \,\middle|\, \begin{matrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{CSP}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{matrix} \,\middle|\, \begin{matrix}\text{patient}\\ \text{has}\\ \text{undiagnosed}\\ \text{CSP}\end{matrix}\right)$$

$$+ P\begin{pmatrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{CSP}\end{pmatrix} \cdot P\left(\begin{matrix}\text{patient}\\ \text{has}\\ \text{diagnosed}\\ \text{CSP}\end{matrix} \,\middle|\, \begin{matrix}\text{Palsy}\\ \text{results}\\ \text{from}\\ \text{CSP}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{patient with}\\ \text{3rd Nerve}\\ \text{Palsy}\\ \text{dies}\end{matrix} \,\middle|\, \begin{matrix}\text{patient}\\ \text{has}\\ \text{diagnosed}\\ \text{CSP}\end{matrix}\right) =$$

$$= 98\% \cdot 0 + 1\% \cdot 5\% \cdot 2\% + 1\% \cdot 5\% \cdot 50\% + 1\% \cdot 95\% \cdot 20\% = 0.00216,$$

meaning that 2,160 deaths will occur for every million of 3rd Nerve Palsy cases, if patients are diagnosed with MRA scan and given appropriate treatment for their diagnosis, or approximately 1 in 463.

(3). Assume that magnetic resonance angiogaphy (MRA) has general 5% error rate (meaning both a non-detection of an existing disease, as well as a false detection of a non-existing disease). Suppose a 3rd Nerve Palsy patient has positive MRA result for aneurysm. First guess, and then find using the Bayes formula, the probability that the patient actually has aneurysm. Are you surprised?

Space for your solution:

$$P\left(\begin{array}{c}\text{patient}\\\text{has}\\\text{aneurysm}\end{array}\middle|\begin{array}{c}\text{positive}\\\text{MRA}\\\text{scan for}\\\text{aneurysm}\end{array}\right) = \boxed{\text{Bayes' formula}} =$$

$$= \frac{P\left(\begin{array}{c}\text{patient}\\\text{has}\\\text{aneurysm}\end{array}\right)\cdot P\left(\begin{array}{c}\text{positive}\\\text{MRA}\\\text{scan for}\\\text{aneurysm}\end{array}\middle|\begin{array}{c}\text{patient}\\\text{has}\\\text{aneurysm}\end{array}\right)}{P\left(\begin{array}{c}\text{patient}\\\text{has}\\\text{aneurysm}\end{array}\right)\cdot P\left(\begin{array}{c}\text{positive}\\\text{MRA}\\\text{scan for}\\\text{aneurysm}\end{array}\middle|\begin{array}{c}\text{patient}\\\text{has}\\\text{aneurysm}\end{array}\right)+P\left(\begin{array}{c}\text{patient}\\\text{has no}\\\text{aneurysm}\end{array}\right)\cdot P\left(\begin{array}{c}\text{positive}\\\text{MRA}\\\text{scan for}\\\text{aneurysm}\end{array}\middle|\begin{array}{c}\text{patient}\\\text{has no}\\\text{aneurysm}\end{array}\right)} =$$

$$= \frac{1\% \cdot 95\%}{1\% \cdot 95\% + 99\% \cdot 5\%} \approx 0.161 = 16.1\%.$$

Problem 3. A grain mill manufactures 100-pound bags of flour for sale in restaurant-supply warehouses. Historically, the weights of bags of flour manufactured at the mill were normally distributed with a mean $\mu = 100$ pounds and a standard deviation $\sigma = 15$ pounds.

(1). What is the probability that the weight of a randomly selected bag of flour falls between 94 and 106 pounds? Use the table of Standard Normal Distribution included at the end of this exam.

Space for your solution:

The z-score $\frac{x-\mu}{\sigma}$ becomes $\frac{94-100}{15} = -0.4$ for $x = 94$ and $\frac{106-100}{15} = 0.4$ for $x = 106$. Using the table of standard normal distribution, we get:

$$P(94 < x < 106) = P(-0.4 < z < 0.4) = 2 \cdot P(0 < z < 0.4) = 2 \cdot 0.1554 = 0.3108.$$

(2). If samples of 36 bags are taken, what is the $\sigma_{\bar{X}}$, the standard error of the mean?

Space for your solution:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{36}} = 2.5.$$

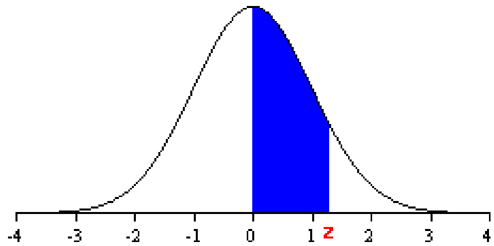
(3). What is the probability that a sample of 36 bags of flour has a mean weight between 94 and 106 pounds?

Space for your solution:

In the manner analogous to finding the z-score for the single bag of flour, the z-score for the sample $\frac{\bar{X}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$ becomes $\frac{94-100}{2.5} = -2.4$ for $x = 94$ and $\frac{106-100}{2.5} = 2.4$ for $x = 106$. Using the table of standard normal distribution, we get:

$$P(94 < x < 106) = P(-2.4 < z < 2.4) = 2 \cdot P(0 < z < 2.4) = 2 \cdot 0.4918 = 0.9836.$$

Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.10	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.20	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.30	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.40	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.50	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.60	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.70	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.80	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.90	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.00	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.10	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.20	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.30	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.40	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.50	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.60	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.70	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.80	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.90	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.00	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.10	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.20	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.30	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.40	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.50	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.60	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.70	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.80	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.90	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.00	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990