

Suffolk County Community College
Michael J. Grant Campus
Department of Mathematics

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MAT 106:
Mathematics for Health Science
Final Exam: Solutions and Answers

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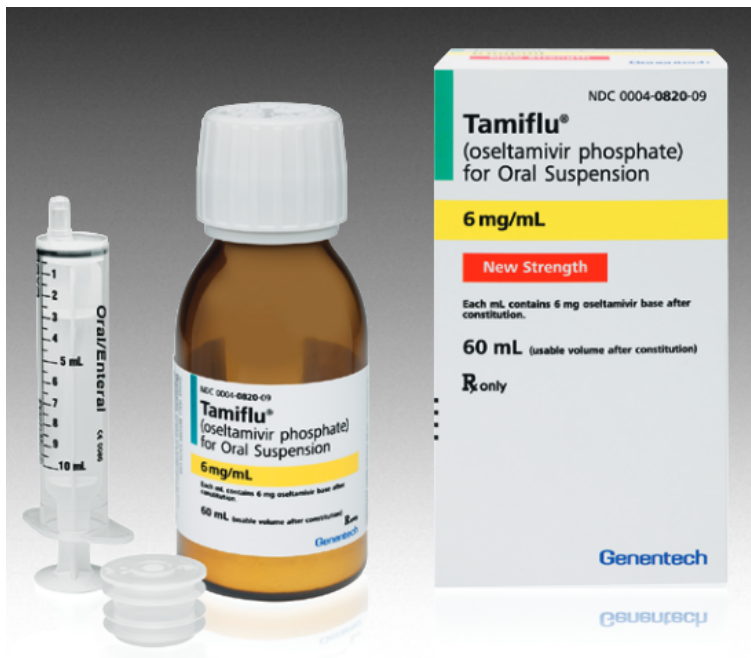
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Problem 1. Tamiflu (generic name: oseltamivir phosphate) is an oral anti-viral drug approved for the treatment of acute, uncomplicated influenza in patients 2 weeks of age and older whose flu symptoms have not lasted more than two days. It belongs to the class of drugs called *neuraminidase inhibitors*. These drugs work by stopping the spread of the influenza (flu) virus in the body, and help shorten the time you have flu symptoms, by about one third on average.

The full course of recommended treatment consists of 10 doses, taken twice daily in the course of 5 days. Each dose depends on the weight and age of the patient, as outlined in the following table:

Patient's Age	Patient's Weight	Single Dose
2 weeks to 1 year	any	3 mg/kg
1 to 12 years	15 kg or less	30 mg
1 to 12 years	15.1 to 23 kg	45 mg
1 to 12 years	23.1 to 40 kg	60 mg
any	40.1 kg or more	75 mg

Suppose you have the drug available in the form of oral suspension:



(1). Determine the single dose needed for a 5 year old child that weighs 20 kg. (Remember that the “dose” is what you directly administer, so it must be in the units you can use in the field.)

Space for your solution:

The amount of pure drug that needs to be administered can be found in the above table and is 45 mg. To convert it to mL of available solution:

	mg	mL
Concentration	6	1
Dose	45	x

(The “concentration” row is taken from the label.) Therefore $x = \frac{45 \cdot 1}{6}$ mL = 7.5 mL.

(2). Determine the single dose needed for a 6 month old baby that weighs 6.5 kg.

Space for your solution:

First, determine the weight of pure drug that needs to be administered, based on the weight of the patient:

Pure Drug	mg	kg
Concentration	3	1
Dose	x	6.5

Therefore, the weight of the pure drug ordered was $x = \frac{3 \cdot 6.5}{1}$ mg = 19.5 mg. Next, convert the weight of pure drug into volume of solution:

Solution	mg	mL
Concentration	6	1
Dose	19.5	y

Thus $y = \frac{19.5 \cdot 1}{6}$ mL = 3.25 mL.

Problem 2. Sickle cell anaemia is an autosomal recessive disorder. It affects erythrocytes (the red blood cells that transport oxygen). Individuals with two normal alleles have normal erythrocytes, but are easily infected with the malaria.

Those who have two defective alleles suffer from the anaemia. Their erythrocytes develop abnormally and may collapse when deoxygenated. However, malaria parasite cannot grow in those abnormal erythrocytes. Therefore people with anaemia are protected from malaria, but suffer from the effects of the erythrocyte defect.

Those who are heterozygous (i.e. are carriers: have one normal and one defective allele) have some sickling of erythrocytes, but do not suffer any ill effects from it, except when severely dehydrated or deprived of oxygen. In addition, malaria parasite cannot reproduce well within these their partially defective erythrocytes. Thus, heterozygous individuals tend to reproduce at a higher rate than those who have one of the two homozygous genotypes.

Compute all probabilities with at least four digits after the decimal.

(1). In a particular family, one parent is healthy and another one has sickle cell anaemia.

They had a child who also suffers from the anaemia. What is the probability that their next child will have sickle cell anaemia?

Space for your solution:

Based on the fact that these two parents have a child who suffers from sickle cell anemia, we can conclude that the healthy parent is a carrier of one sickle cell anemia allele. Therefore

$$P(\text{next child has sickle cell anemia}) = P\left(\begin{array}{c} \text{carrier parent} \\ \text{gives the} \\ \text{defective allele} \\ \text{to the next child} \end{array}\right) = \frac{1}{2}.$$

(2). Sickle cell anaemia is estimated to occur in 1 in 500 African Americans. What are the frequencies of the normal and defective sickle cell anaemia alleles in the African American population?

Space for your solution:

Since the sickle cell anaemia is the recessive trait,

$$P\left(\begin{array}{c} \text{defective} \\ \text{allele} \end{array}\right) = \sqrt{P\left(\begin{array}{c} \text{sickle cell} \\ \text{anaemia} \end{array}\right)} = \sqrt{\frac{1}{500}} \approx 0.0447.$$

By the formula of probability of the complement event,

$$P\left(\begin{array}{c} \text{normal} \\ \text{allele} \end{array}\right) = 1 - P\left(\begin{array}{c} \text{defective} \\ \text{allele} \end{array}\right) \approx 1 - 0.0447 = 0.9553.$$

(3). Using the information from the previous subproblem, determine the probability of an African American to be a (healthy) carrier of sickle cell anaemia allele.

Space for your solution:

Consider the Punnett square for the various sickle cell anaemia genotypes and their probabilities, for a child born to African American parents. Denote the normal sickle cell allele as S and the defective one as s.

	Father gave S, 0.9553	Father gave s, 0.0447
Mother gave S, 0.9553	SS, 0.9126	sS, 0.0427
Mother gave s, 0.0447	Ss, 0.0427	ss, 0.002

(The entries in the above table are rounded, and therefore approximate.) Therefore

$$\begin{aligned}
 P\left(\begin{array}{c} \text{child is} \\ \text{heterozygous} \end{array}\right) &= \\
 P\left(\left(\begin{array}{c} \text{child has} \\ \text{genotype} \\ \text{sS} \end{array}\right) \cup \left(\begin{array}{c} \text{child has} \\ \text{genotype} \\ \text{Ss} \end{array}\right)\right) &= P\left(\begin{array}{c} \text{child has} \\ \text{genotype} \\ \text{sS} \end{array}\right) + P\left(\begin{array}{c} \text{child has} \\ \text{genotype} \\ \text{Ss} \end{array}\right) = \\
 &\approx 0.0427 + 0.0427 = 0.0854.
 \end{aligned}$$

(4). In a particular family, one parent is a healthy African American and another one has sickle cell anaemia. Determine the probability that their child will have sickle cell anaemia.

Space for your solution:

Using the results of the previous subproblem and the formula of total probability,

$$\begin{aligned}
 P(\text{child has sickle cell anaemia}) &= \\
 P\left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is not a} \\ \text{carrier} \end{array}\right) \cdot P\left(\begin{array}{c} \text{child has} \\ \text{sickle cell} \\ \text{anaemia} \end{array} \middle| \begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is not a} \\ \text{carrier} \end{array}\right) &+ P\left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is a} \\ \text{carrier} \end{array}\right) \cdot P\left(\begin{array}{c} \text{child has} \\ \text{sickle cell} \\ \text{anaemia} \end{array} \middle| \begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is a} \\ \text{carrier} \end{array}\right) \\
 = P\left(\begin{array}{c} \text{African} \\ \text{American} \\ \text{is a carrier} \end{array} \middle| \begin{array}{c} \text{African} \\ \text{American} \\ \text{is healthy} \end{array}\right) \cdot \frac{1}{2} &= \frac{P\left(\begin{array}{c} \text{African} \\ \text{American} \\ \text{is a carrier} \end{array}\right)}{2 \cdot P\left(\begin{array}{c} \text{African} \\ \text{American} \\ \text{is healthy} \end{array}\right)} \approx \frac{0.0854}{2 \cdot (0.0854 + 0.9126)} \approx 0.0428.
 \end{aligned}$$

(5). In a particular family, one parent is a healthy African American and another one has sickle cell anaemia. They had a child who is healthy. Determine the probability of their next child having sickle cell anaemia.

Space for your solution:

$P(\text{healthy parent is a carrier} | \text{child does not have sickle cell anaemia}) = \boxed{\text{Bayes' formula}} =$

$$\begin{aligned}
 & P \left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is a} \\ \text{carrier} \end{array} \right) \cdot P \left(\begin{array}{c|c} \text{child} & \text{healthy} \\ \text{does not have} & \text{parent} \\ \text{sickle cell} & \text{is a} \\ \text{anaemia} & \text{carrier} \end{array} \right) \\
 \hline
 & P \left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is not a} \\ \text{carrier} \end{array} \right) \cdot P \left(\begin{array}{c|c} \text{child} & \text{healthy} \\ \text{does not have} & \text{parent} \\ \text{sickle cell} & \text{is not a} \\ \text{anaemia} & \text{carrier} \end{array} \right) + P \left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is a} \\ \text{carrier} \end{array} \right) \cdot P \left(\begin{array}{c|c} \text{child} & \text{healthy} \\ \text{does not have} & \text{parent} \\ \text{sickle cell} & \text{is a} \\ \text{anaemia} & \text{carrier} \end{array} \right) \\
 & \qquad \qquad \qquad P \left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is a} \\ \text{carrier} \end{array} \right) \cdot \frac{1}{2} \\
 & = \frac{P \left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is not a} \\ \text{carrier} \end{array} \right) \cdot 1 + P \left(\begin{array}{c} \text{healthy} \\ \text{parent} \\ \text{is a} \\ \text{carrier} \end{array} \right) \cdot \frac{1}{2}}{P \left(\begin{array}{c|c} \text{African} & \text{African} \\ \text{American} & \text{American} \\ \text{is a carrier} & \text{is healthy} \end{array} \right) \cdot \frac{1}{2}} \\
 & = \frac{P \left(\begin{array}{c|c} \text{African} & \text{African} \\ \text{American} & \text{American} \\ \text{is not a carrier} & \text{is healthy} \end{array} \right) + P \left(\begin{array}{c|c} \text{African} & \text{African} \\ \text{American} & \text{American} \\ \text{is a carrier} & \text{is healthy} \end{array} \right) \cdot \frac{1}{2}}{\approx \frac{0.0428}{0.9144 + 0.0428} \approx 0.0447.}
 \end{aligned}$$

$$P \left(\begin{array}{c} \text{next child} \\ \text{has sickle cell} \\ \text{anaemia} \end{array} \right) = P \left(\begin{array}{c} \text{healthy parent} \\ \text{is a carrier} \end{array} \right) \cdot \frac{1}{2} \approx 0.0447 \cdot \frac{1}{2} \approx 0.0224.$$