Suffolk County Community College
Michael J. Grant Campus
Department of Mathematics

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MAT 106
Mathematics for Health Science

Final Exam: Solutions and Answers

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Problem 1. Midazolam hydrochloride is available in a bottle with the following label:

(1). What weight of the pure drug is contained in the whole bottle?

Space for your solution:

From the drug label:

	mg of pure drug	**mL of solution**
Concentration of the solution	2	1
Whole bottle	x	118

This gives $x = \frac{2 \cdot 118}{1}$ mg $= 236$ mg.

(2). Determine the needed dose of the medication, if 15 mg of the drug is ordered.

Space for your solution:

Using similar table:

	mg of pure drug	**mL of solution**
Concentration of the solution	2	1
Ordered	15	x

This gives $x = \frac{15 \cdot 1}{2}$ mg $= 7.5$ mL.

(3). How many full doses (as computed in the previous sub-problem) are contained in this bottle?

Space for your solution:

As we computed in the previos problem, a single dose is 7.5 mL. Since the whole bottle is 118 mL, the bottle contains $\frac{118}{7.5} = 15.7333\ldots$, i.e. 15 full doses of the medication.

Problem 2. This problem will introduce you to the *Simpson's Paradox*.

1314 women took part in a study [1] of thyroid disease that was conducted in 1972-1974 in Newcastle, United Kingdom. A follow-up study of the same subjects [2] took place nearly thirty years later.

(1). The subjects of the study were classified according to their smoking habits (current smokers at the time of the original 1970's study or those who never smoked) and according to their survival status 20 years after the original study. The outcomes are summarized in the following table:

	Smoker	**Non-smoker**
Dead	139	230
Alive	443	502

Based on this table, determine if smoking has positive or negative effect on survival.
Hint: compute and compare the conditional probabilities:

$$P(\text{Alive}|\text{Smoker})$$

$$P(\text{Alive}|\text{Non-smoker})$$

Space for your solution:

$$P(\text{Alive}|\text{Smoker}) = \frac{443}{139+443} \approx 76\%$$
$$P(\text{Alive}|\text{Non-smoker}) = \frac{502}{230+502} \approx 69\%$$

Thus it seems that smoking has positive effect on survival.

[1] W. M. G. Tunbridge, D. C. Evered, D. Appleton, M. Brewis, F. Clark
"The Spectrum of Thyroid Disease in a Community: The Whickham Survey",
Clinical Endocrinology, Volume 7, Issue 6, December 1977, Pages 481-493
`http://onlinelibrary.wiley.com/doi/10.1111/j.1365-2265.1977.tb01340.x/abstract`

[2] David R. Appleton, Joyce M. French and Mark P. J. Vanderpump
"Ignoring a Covariate: An Example of Simpson's Paradox",
The American Statistician, Volume 50, Number 4, November 1996, Pages 340-341
`http://www.jstor.org/stable/2684931?seq=1#page_scan_tab_contents`

(2). The subjects were further classified according to their age at the time of the original study. The outcomes for women aged 18 to 64 are summarized in this table:

Age 18 to 64	**Smoker**	**Non-smoker**
Dead	97	65
Alive	436	474

Determine if smoking has positive effect on survival of women in this age group.

Space for your solution:

$$P(\text{Alive}|\text{Smoker aged 18 to 64}) = \frac{436}{97+436} \approx 82\%$$

$$P(\text{Alive}|\text{Non-smoker aged 18 to 64}) = \frac{474}{65+474} \approx 88\%$$

Thus it seems that smoking has negative effect on survival within the 18 to 64 years of age group.

(3). The outcomes for women aged 65 and above are summarized in this table:

Age 65 and above	**Smoker**	**Non-smoker**
Dead	42	165
Alive	7	28

Determine if smoking has positive effect on survival of women in this age group.

Space for your solution:

$$P(\text{Alive}|\text{Smoker aged 18 to 64}) = \frac{7}{42+7} \approx 14.3\%$$

$$P(\text{Alive}|\text{Non-smoker aged 18 to 64}) = \frac{28}{165+28} \approx 14.5\%$$

Thus it seems that smoking has (a very slight) negative effect on survival within the 65 years and above age group.

(4). What conclusion can you draw from this consideration: does smoking improve or harm survival chances? If smoking is beneficial, why it is not shown by the analysis of age groups? If smoking is harmful, why does it contradict the outcome for the combined analysis (that ignores age)?

Space for your solution:

The main predictor of survival over the 20 year period is age. Among the women studied, younger women (who have lower mortality) had higher percentage of smokers, and older women (who have higher mortality) had lower percentage of smokers. In the aggregated analysis, the stronger effect of age on survival masked the effect of smoking status, creating the illusion of beneficial effect of smoking on survival. The smokers in the aggregated sample died at a lower rate not because smoking was beneficial, but because smokers tended to be younger. When the two age groups are analyzed separately, the effect of smoking is isolated from the effect of age, and the negative effect of smoking on survival becomes apparent.

Problem 3. The sensation of taste is a combination of five basic tastes: sweet, bitter, sour, salty, and glutamic (called *umami* by Japanese). Synthetic compound phenylthiocarbamide (PTC) is used to study the genetics of bitter perception. PTC tastes very bitter to most persons. The inability to taste PTC is controlled by a single autosomal recessive gene, called TAS2R38. In the white American population, about 70% can taste PTC, while 30% cannot (are non-tasters). Assume that the white American population is in the state of Hardy-Weinberg equilibrium with respect to the taster (T) and nontaster (t) alleles. The taster trait is dominant and non-taster — recessive.

(1). Find the frequency of the of the taster (T) and nontaster (t) alleles in the white American population.

Space for your solution:

Denote the frequency of allele T as x and the frequency of allele t as y. Then we will get the following Punnett square for a white American child:

	Father gave T, x	**Father gave t,** y
Mother gave T, x	genotype TT: tester x^2	genotype tT: tester $x \cdot y$
Mother gave t, y	genotype Tt: tester $x \cdot y$	genotype tt: non-taster $y^2 = 30\%$

Therefore, we can find y by solving the equation $y^2 = 30\%$, and we get that $y = \sqrt{30\%} = 55\%$. Since $x + y = 1$, we can find $x = 1 - y = 100\% - 55\% = 45\%$.

(2). Find the probability of each TAS2R38 genotype in the white American population.

Space for your solution:

Now that we know the the allele frequencies x and y, we can compute the probabilities in the above Punnett square:

	Father gave T, x	**Father gave t,** y
Mother gave T, x	genotype TT: tester 20%	genotype tT: tester 25%
Mother gave t, y	genotype Tt: tester 25%	genotype tt: non-taster 30%

(3). Find the probability that one taster and one non-taster white American parent give birth to a taster child.

Space for your solution:

Assume that father is tester and mother is non-tester.

$$P\begin{pmatrix}\text{child is}\\\text{non-tester}\end{pmatrix}\boxed{=\text{non-tester trait is recessive}=}P\begin{pmatrix}\text{child has}\\\text{two alleles t}\end{pmatrix}=$$

$$P\left(\begin{pmatrix}\text{child}\\\text{got}\\\text{allele t}\\\text{from}\\\text{father}\end{pmatrix}\&\begin{pmatrix}\text{child}\\\text{got}\\\text{allele t}\\\text{from}\\\text{mother}\end{pmatrix}\right)\boxed{=\text{product rule}=}P\begin{pmatrix}\text{child}\\\text{got}\\\text{allele t}\\\text{from}\\\text{father}\end{pmatrix}\cdot P\begin{pmatrix}\text{child}\\\text{got}\\\text{allele t}\\\text{from}\\\text{mother}\end{pmatrix}=$$

$$P\begin{pmatrix}\text{child got}\\\text{allele t}\\\text{from father}\end{pmatrix}\cdot 1=P\begin{pmatrix}\text{child got}\\\text{allele t}\\\text{from father}\end{pmatrix}\boxed{=\text{formula of total probability}=}$$

$$P\begin{pmatrix}\text{father is}\\\text{heterozygous}\\\text{tester}\end{pmatrix}\cdot P\left(\begin{matrix}\text{child got}\\\text{allele t}\\\text{from father}\end{matrix}\middle|\begin{matrix}\text{father is}\\\text{heterozygous}\\\text{tester}\end{matrix}\right)+$$
$$P\begin{pmatrix}\text{father is}\\\text{homozygous}\\\text{tester}\end{pmatrix}\cdot P\left(\begin{matrix}\text{child got}\\\text{allele t}\\\text{from father}\end{matrix}\middle|\begin{matrix}\text{father is}\\\text{homozygous}\\\text{tester}\end{matrix}\right)=$$

$$P\begin{pmatrix}\text{father is}\\\text{heterozygous}\\\text{tester}\end{pmatrix}\cdot\frac{1}{2}+P\begin{pmatrix}\text{father is}\\\text{homozygous}\\\text{tester}\end{pmatrix}\cdot 0=P\begin{pmatrix}\text{father is}\\\text{heterozygous}\\\text{tester}\end{pmatrix}\cdot\frac{1}{2}=$$

$$P\left(\begin{matrix}\text{white}\\\text{American is}\\\text{heterozygous}\end{matrix}\middle|\begin{matrix}\text{white}\\\text{American}\\\text{is tester}\end{matrix}\right)\cdot\frac{1}{2}=\frac{P\begin{pmatrix}\text{white}\\\text{American is}\\\text{heterozygous}\end{pmatrix}}{P\begin{pmatrix}\text{white}\\\text{American}\\\text{is tester}\end{pmatrix}}\cdot\frac{1}{2}=\frac{25\%+25\%}{20\%+25\%+25\%}\cdot\frac{1}{2}=35\%.$$

Thus

$$P\begin{pmatrix}\text{child is}\\\text{tester}\end{pmatrix}=1-P\begin{pmatrix}\text{child is}\\\text{non-tester}\end{pmatrix}=65\%.$$