Suffolk County Community College
Michael J. Grant Campus
Department of Mathematics

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MAT 120
College Algebra and Trigonometry

Final Exam: Solutions and Answers

Instructor:
Name: Alexander Kasiukov
Office: Suffolk Federal Credit Union Arena, Room A-109
Phone: (631) 851-6484
Email: kasiuka@sunysuffolk.edu
Web Site: `http://kasiukov.com`

Problem 1. Solve the equation $\ln(x) - 3 = \ln(x+2)$.

Space for your solution:

$$\ln(x) - 3 = \ln(x+2) \quad \Leftrightarrow \quad \ln(x) - \ln(x+2) = 3$$

$$\Leftrightarrow \begin{cases} \ln\left(\dfrac{x}{x+2}\right) = 3 \\ x > 0 \\ x+2 > 0 \end{cases} \quad \Leftrightarrow \begin{cases} e^{\ln\left(\frac{x}{x+2}\right)} = e^3 \\ x > 0 \end{cases} \quad \Leftrightarrow \begin{cases} \dfrac{x}{x+2} = e^3 \\ x > 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = e^3(x+2) \\ x > 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = \dfrac{2e^3}{1-e^3} \\ x > 0 \end{cases} \quad \Leftrightarrow \quad x \in \emptyset.$$

Problem 2. Solve the equation $(\log_7 x) - 1 = \log_7(x+1)$.

Space for your solution:

$$(\log_7 x) - 1 = \log_7(x+1) \quad \Leftrightarrow \quad 7^{(\log_7 x)-1} = 7^{\log_7(x+1)} \quad \Leftrightarrow \quad \frac{7^{(\log_7 x)}}{7^1} = 7^{\log_7(x+1)}$$

$$\Leftrightarrow \begin{cases} \frac{x}{7} = x+1 \\ x > 0 \\ x+1 > 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = 7x+7 \\ x > 0 \end{cases} \quad \Leftrightarrow \begin{cases} -6x = 7 \\ x > 0 \end{cases} \quad \Leftrightarrow \begin{cases} x = -\frac{7}{6} \\ x > 0 \end{cases}$$

$$\Leftrightarrow \quad x \in \emptyset.$$

Problem 3. Solve the equation $5^{2x} = \frac{1}{3^{x-1}}$.

Space for your solution:

$$5^{2x} = \tfrac{1}{3^{x-1}} \quad \Leftrightarrow \quad 5^{2x} = 3^{-(x-1)} \quad \Leftrightarrow \quad 25^x = 3^{-x}3^1 \quad \Leftrightarrow \quad 25^x 3^x = 3$$

$$\Leftrightarrow \quad 75^x = 3 \quad \Leftrightarrow \quad \log_{75} 75^x = \log_{75} 3 \quad \Leftrightarrow \quad x = \log_{75} 3.$$

Problem 4. Solve the equation $2^{x-2} = 2^x + 3$.

Space for your solution:

$$2^{x-2} = 2^x + 3 \quad \Leftrightarrow \quad \frac{2^x}{2^2} = 2^x + 3 \quad \Leftrightarrow \quad 2^x = 4 \cdot 2^x + 12 \quad \Leftrightarrow \quad -3 \cdot 2^x = 12 \quad \Leftrightarrow \quad 2^x = -4$$

$$\Leftrightarrow \quad x \in \emptyset.$$

Problem 5. Consider the system of linear equations: $\begin{cases} x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1 \\ 2x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 = 2 \\ 3x_1 + 2x_2 - 4x_3 - 3x_4 - 9x_5 = 3 \end{cases}$

(1). Perform the downward Gauss-Jordan method on the augmented matrix of the above system.

Space for your solution:

The augmented matrix of the above system is

$$\left[\begin{array}{rrrrr|r} 1 & 1 & -2 & 1 & 3 & 1 \\ 2 & -1 & 2 & 2 & 6 & 2 \\ 3 & 2 & -4 & -3 & -9 & 3 \end{array}\right]$$

Consider the first column. The upper-most leader is in the first row and that leader is already 1. Therefore we start by zeroing out the entries under the first row leader. Add to the second row -2 times the first row; add to the third row -3 times the first row. These operations yield

$$\left[\begin{array}{rrrrr|r} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & -3 & 6 & 0 & 0 & 0 \\ 0 & -1 & 2 & -6 & -18 & 0 \end{array}\right]$$

Consider the second column. The upper-most leader is -3 and that leader is in the second row. Divide the second row by -3 to get its leader equal 1. (At this point it is also permissible to interchange the third and second rows to avoid division by -3.) We get

$$\left[\begin{array}{rrrrr|r} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & -1 & 2 & -6 & -18 & 0 \end{array}\right]$$

Next, zero out the entry under the leader in the second column. Add to the third row (1 times) the second row. This operation yields

$$\left[\begin{array}{rrrrr|r} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6 & -18 & 0 \end{array}\right]$$

Consider the third column. It has no leader and therefore we go to the fourth column. The leader of the fourth column is in the third row and that leader is equal to -6. Divide the third row by -6 to get its leader equal 1. We get

$$\left[\begin{array}{rrrrr|r} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{array}\right]$$

Consider the fifth column. It has no leader and therefore we go to the sixth column. The sixth (and the last) column has no leader and therefore we are done.

(2). Obtain the reduced row echelon form of the augmented matrix of the original linear system (i.e. perform the upward Gauss-Jordan method on the augmented matrix, obtained in the previous subproblem).

Space for your solution:

We have the matrix

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 1 & 3 & 1 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{array}\right]$$

The third row leader is in the fourth column. We will zero out all entries above that leader. Add to the first row -1 times the third row. We will get

$$\left[\begin{array}{ccccc|c} 1 & 1 & -2 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{array}\right]$$

The second-row leader is in the second column. We will zero out all entries above that leader. Add to the first row -1 times the second row. We will get

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 \end{array}\right]$$

The above matrix is in the reduced row echelon form.

(3). Find a particular solution of the original system of linear equations and a system of fundamental solutions of the associated homogeneous system.

Space for your solution:

The column of constants of the reduced row echelon matrix of the system has no leader. Therefore the system has at least one solution. The columns of the coefficients of x_3 and of x_5 have no leaders. Therefore we can express x_1, x_2 and x_4 in terms of x_3 and x_5: $\begin{cases} x_1 = 1 \\ x_2 = 2x_3 \\ x_4 = -3x_5 \end{cases}$. Introduce fake variables $s = x_3$ and $t = x_5$:

$$\begin{cases} x_1 = 1 \\ x_2 = 2s \\ x_3 = s \\ x_4 = -3t \\ x_5 = t \end{cases} \Leftrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\substack{\text{this vector is a} \\ \text{particular solution} \\ \text{of the original system}}} + \underbrace{s \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3 \\ 1 \end{bmatrix}}_{\substack{\text{these two vectors form a} \\ \text{system of fundamental solutions} \\ \text{of the associated homogeneous system}}}$$