Suffolk County Community College

Michael J. Grant Campus

Department of Mathematics

Spring 2025

MAT 142
Calculus with Analytic Geometry II

Final Exam

Instructor:

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Please print the requested information in the spaces provided:

Student:

Name:

Student Id:

Email: *include to receive the final grade via email* ONLY *if you are not getting email updates*

- *Notes and books are permitted on this exam.*
- *Graphing calculators, smartwatches, computers, cell phones and any other communication-capable devices are prohibited. Their mere presence in the open (even without use) is a sufficient reason for an immediate dismissal from this exam with a failing grade.*
- *You will not receive full credit if there is no work shown, even if you have the right answer. Please don't attach additional pieces of paper: if you run out of space, please ask for another blank final.*

Problem 1. Compute the integral

$$\int \frac{\sin\left(\sqrt{x}\right)}{\sqrt{x}} \quad \mathtt{d}x.$$

Space for your solution:

Problem 2. Compute the integral

$$\int \cos\left(\sqrt{x}\right) \quad \mathtt{d}x.$$

Space for your solution:

Problem 3. Find the

$$\int \frac{2x^3 - 5x^2 + 7}{x^2 - 4x + 4} \ \mathtt{d}x.$$

Space for your solution:

Problem 4. Consider the function $f(x) = \sin(x)$.

(1). Find the formula for

$$\left(\frac{\mathtt{d}}{\mathtt{d}x}\right)^i f(x)$$

with arbitrary x for $i = 0, 1, 2, 3, 4, 5$.

Space for your solution:

(2). Which point (or points) from the domain of the function f would be a good choice for the center of a Taylor polynomial for f, and why?

Space for your solution:

(3). Explicate the Taylor polynomial approximation

$$f(x) = \sum_{i=0}^{n} \frac{1}{i\,!} \cdot \left(\left(\frac{d}{d\,t}\right)^{i} \Bigg|_{t=x_0} f(t) \right) \cdot (x - x_0)^i + \frac{1}{n\,!} \int_{t=x_0}^{x} \left(\left(\frac{d}{d\,t}\right)^{n+1} f(t) \right) \cdot (x-t)^n \, d\,t$$

for $f(x) = \sin(x)$, $n = 5$, and x_0 selected in the previous sub-problem. The final answer must contain neither the symbols of differentiation, nor the sigma notation.

Space for your solution:

(4). Explicate the Taylor polynomial approximation

$$f(x) = \sum_{i=0}^{n} \frac{1}{i\,!} \cdot \left(\left(\frac{d}{d\,t}\right)^{i} \Bigg|_{t=x_0} f(t) \right) \cdot (x - x_0)^i + \frac{1}{n\,!} \int_{t=x_0}^{x} \left(\left(\frac{d}{d\,t}\right)^{n+1} f(t) \right) \cdot (x-t)^n \, d\,t$$

for $f(x) = \sin(x)$ and arbitrary n at the x_0 selected previously. The final answer must contain the sigma notation, but may have the differentiation symbol only in the error term.

Space for your solution:

(5). Explicate the Taylor polynomial approximation

$$f(x) = \sum_{i=0}^{n} \frac{1}{i\,!} \cdot \left(\left(\frac{d}{d\,t} \right)^{i} \Bigg|_{t=x_0} f(t) \right) \cdot (x - x_0)^i + \frac{1}{n\,!} \int_{t=x_0}^{x} \left(\left(\frac{d}{d\,t} \right)^{n+1} f(t) \right) \cdot (x - t)^n \, d\,t$$

for $f(x) = \sin(x)$ and $n = 1000$ at the x_0 selected previously. The final answer must contain the sigma notation, but may have the differentiation symbol only in the error term.

Space for your solution:

(6). For the degree n Taylor polynomial approximation of $\sin(x)$, find a computable estimate of the error that has the limit 0 as $n \to +\infty$.

Space for your solution:

(7). Estimate $\sin(1)$ with guaranteed precision $\varepsilon = 0.01$.

Space for your solution: